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Spherical Symmetry and Mass-Energy in Higher Dimensions

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The components of the Einstein tensor and other relations are given for a spherically symmetric metric in null coordinates in higher dimensions. These relations are particularly relevant to the study of gravitational collapse of a perfect fluid with heat flow but without viscosity, where the exterior space cannot be considered as vacuum and matching to Schwarzschild space-time is not suitable. The analysis generalizes to higher dimensions work of Cahill and McVittie in 4D space-time. Using the expression for the "mass function," it is observed that pressure vanishes at the boundary of the distribution for a perfect fluid in the higher-dimensional case also, but the same is not true when heat flow is considered.

1. INTRODUCTION

The pioneering work of Oppenheimer and Snyder (1939) over 50 years have greatly enhanced our understanding of the gravitational collapse of any starlike spherical distribution of matter, particularly at its late stage of evolution. The situation analyzed by them is too simplistic in the sense that pressure, rotation, radiation, asymmetries, and other important quantum effects are neglected. There has been of late a resurgence of interest in models where the energy-momentum tensor contains heat flow and radiation in addition to a perfect fluid, because a nonstatic starlike object in general would be radiating energy and may contain charges as well (Shvartsman, 1971). Following the detection of QSOs and other extragalactic sources and their colossal energy requirements, Hoyle and Fowler (1963) suggested a theory of hot, convective supermassive stars where general relativistic effects can no longer be neglected. The most appropriate metric for studying this

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radiation is the one given by Bondi *et al.* (1962), where radiation coordinates are introduced.

On the other hand, the long-time goal of a unification of gravity with other forces in nature continues to remain elusive in quantum field theory. Most recent efforts in this search have been directed at studying theories in which the dimension of the space-time is greater than the $(3+1)$ of the world that we observe. Further, the advance of work on supergravity in 11 dimensions and superstrings in 10 dimensions indicates that the multidimensionality of space is apparently a fairly adequate reflection of the dynamics of interaction over distances $r \ll 10^{-16}$ cm where the unification of all types of forces is possible (Witten, 1984).

Recently there has been a proliferation of articles on higher-dimensional space time both in localized and cosmological domains (Myers and Perry, 1986; Chatterjee, 1987; Banerjee *et al.*, 1990; Chatterjee *et al.*, 1990*a*) [also see Emelyanov *et al.* (1986) for an excellent review]. For a localized distribution all the solutions so far obtained are arbitrary-dimensional generalizations of the usual Schwarzschild or Kerr solutions. In a recent communication Liddle *et al.* (1990) studied the consequences of the extra dimensions on the structure of neutron stars and showed that the presence of the extra dimensions reduces the maximum mass of the star. The situation may, however, improve if a more realistic equation of state is considered. However, to our knowledge, nonstatic models of the same type have not been considered so far within the standard Einstein theory. So in view of the renewed interest in higher-dimensional theories as well as the need of taking into account radiation from localized bodies (particularly starlike objects in astrophysics), we have thought it worthwhile to give in detail the expressions for the Christoffel symbols, Riemann–Christoffel tensor, and components of Einstein's field equations in higher-dimensional radiating coordinates with the hope that they may be useful for those interested in this field. Then, following Cahill and McVittie (1970), we obtain an expression for the mass function in $(n+2)$ dimensions. With the help of the mass function we show that the fluid pressure on the boundary of the spherical distribution vanishes for an adiabatic flow, which, however, is not the case if the fluid contains a heat flow vector. We also get an expression for the effective mass when the energy-momentum tensor contains an electromagnetic field and discuss the possible observational consequences of higher dimensions on this result.

2. MATHEMATICAL FORMULATION

The spherically symmetric line element in null coordinates in $D=(n+2)$ dimensions is

$$ds^2 = e^{2\beta}[(V/r)\,du^2 + 2\,du\,dr] - R^2\,dX_n^2 \tag{1}$$

where

$$dX_n^2 = d\theta_1^2 + \sin^2\phi_1\, d\theta_2^2 + \cdots + \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta \sin^2\theta_{n-1}\, d\theta_n^2$$

We have

$$\beta = \beta(r, u), \qquad V = V(r, u), \qquad R = R(r, u)$$

where u is the retarded time.

Since this type of line element in higher dimensions has not been, to our knowledge, discussed in the literature, we give in some detail the expressions for the nonvanishing components of the Christoffel symbol as also the Riemann–Christoffel tensors in the hope that these may be useful for workers in this field.

For the line element (1) we get

$$g_{ab} = \begin{bmatrix} e^{2\beta}V/r & e^{2\beta} & 0 & 0 & \cdots \\ e^{2\beta} & 0 & 0 & 0 & \cdots \\ 0 & 0 & -R^2 & 0 & \cdots \\ 0 & 0 & 0 & -R^2\sin^2\theta\sin^2\phi & \cdots \end{bmatrix} \tag{2}$$

$$g^{ab} = \begin{bmatrix} 0 & e^{-2\beta} & 0 & 0 & 0 & \cdots \\ e^{-2\beta} & -(V/r)\,e^{-2\beta} & 0 & 0 & 0 & \cdots \\ 0 & 0 & -1/R^2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & (-R^2\sin^2\theta\sin^2\phi)^{-1} & 0 & \cdots \end{bmatrix} \tag{3}$$

The nonvanishing components of Christoffel symbols are

$$\begin{aligned}
\Gamma^0_{00} &= 2\beta_0 - \beta_1 V/r + V/2r^2 - V_1/2r \\
g^{22}\Gamma^0_{22} &= g^{33}\Gamma^0_{33} = \cdots = g^{nn}\Gamma^0_{nn} = -e^{-2\beta}R_1/R \\
\Gamma^1_{00} &= -\beta_0 V/r + \beta_1 V^2/r^2 + V_1 V/2r^2 - V^2/2r^3 + V_0/2r \\
\Gamma^1_{11} &= 2\beta_1 \\
\Gamma^1_{01} &= \beta_1 V/r + V_1/2r - V/2r^2 \\
g^{22}\Gamma^1_{22} &= g^{33}\Gamma^1_{33} = \cdots = g^{nn}\Gamma^1_{nn} = e^{-2\beta}(VR_1/rR - R_0/R) \\
\Gamma^2_{02} &= \Gamma^3_{03} = \cdots = \Gamma^n_{0n} = R_0/R \\
\Gamma^2_{12} &= \Gamma^3_{13} = \cdots = \Gamma^n_{1n} = R_1/R \\
\Gamma^2_{33} &= -\sin\theta\cos\theta \\
\Gamma^3_{44} &= -\sin\phi\cos\phi \\
\Gamma^3_{23} &= \cot\theta \\
\Gamma^4_{24} &= \cot\theta
\end{aligned} \tag{4}$$

We give below the expressions of the Riemann–Christoffel tensors defined by

$$R^i_{jkl}=\Gamma^i_{jl,k}-\Gamma^i_{jk,l}+\Gamma^\gamma_{jl}\Gamma^i_{k\gamma}-\Gamma^\gamma_{jk}\Gamma^i_{\gamma l} \tag{5}$$

which are useful for the derivation of the field equations:

$$R^2_{112}=R^3_{113}=\cdots=R^n_{11n}=R_{11}/R-2\beta_1R_1/R \tag{6}$$

$$g^{22}R^0_{220}=g^{33}R^0_{330}=\cdots=g^{nn}R^0_{nn0}$$
$$=e^{-2\beta}(R_{01}/R-VR_1\beta_1/Rr+VR_1/2Rr^2-V_1R_1/2Rr) \tag{7}$$

$$R^0_{010}=2\beta_{01}-\beta_{11}V/r-\beta_1V_1/r+\beta_1V/r^2+V_1/r^2-V/r^3-V_{11}/2r \tag{8}$$

$$R^i_{jij}=1+2R_0R_1\,e^{-2\beta}-e^{-2\beta}R_1^2V/r \tag{9}$$

$$(i=3,4,5,\ldots,n;j=2,3,4,\ldots,n)$$

$$R^2_{012}=R^3_{013}=\cdots=R^n_{01n}$$
$$=R_{10}/R-VR_1\beta_1/Rr-V_1R_1/2rR+VR_1/2Rr^2 \tag{10}$$

$$R^2_{002}=R^3_{003}=\cdots=R^n_{00n}$$
$$=R_{00}/R+(R_0/R)(V_1/2r-V/2r^2+V\beta_1/r-2\beta_0)$$
$$+(R_1/R)(V\beta_0/r+V^2/2r^3-V^2\beta_1/r^2-V_0/2r-VV_1/2r^2) \tag{11}$$

$$R^1_{001}=2\beta_{01}V/r-\beta_{11}V^2/r^2-\beta_1VV_1/r^2+\beta_1V^2/r^3$$
$$+VV_1/r^3-V^2/r^4-VV_{11}/2r^2 \tag{12}$$

$$R^1_{221}=+e^{-2\beta}(-RR_{01}-VRR_1\beta_1/r-VRR_1/2r^2$$
$$+RR_1V_1/2r+VRR_{11}/r) \tag{13}$$

$$R^1_{101}=2\beta_{01}-V\beta_{11}/r-V_1\beta_1/r+V\beta_1/r^2+V_1/r^2-V/r^3-V_{11}/2r \tag{14}$$

$$R^1_{202}=e^{-2\beta}(RR_{00}-RR_1V_0/2r-RR_{10}V/r-2\beta_0RR_0+V\beta_0RR_1/r$$
$$+RR_0V_1/2r+RR_0V\beta_1/r-RR_0V/2r^2) \tag{15}$$

$$R^2_{102}=R_{01}/R-V\beta_1R_1/rR-V_1R_1/2rR+VR_1/2Rr^2 \tag{16}$$

Here the subscripts 0 and 1 denote differentiation with respect to u and r, respectively.

The components of the Einstein tensor can be found via

$$G_{ij} = R^k_{ijk} - \tfrac{1}{2} g_{ij} g^{\alpha\beta} R^k_{\alpha\beta k} \tag{17}$$

$$G^1_1 = (n/2) g^{22}[(n-1) R^3_{232} + 2R^0_{202}] \tag{18}$$

$$G^2_2 = G^3_3 = \cdots = G^n_n = (n-1) g^{22}(R^0_{202} + R^1_{212} + R^3_{223}) - g^{01} R^0_{010} \tag{19}$$

$$G^0_0 = g^{22}[nR^1_{212} + \tfrac{1}{2} n(n-1) R^3_{232}] \tag{20}$$

$$G^1_0 = g^{\alpha 1} G_{\alpha 0} = -ng^{22} R^1_{202} \tag{21}$$

$$G^0_1 = g^{0\alpha} G_{1\alpha} = -ng^{22} R^0_{212} \tag{22}$$

Using the relations (6)–(16), we finally get

$$G_{11} = n(R_{11}/R - 2\beta_1 R_1/R) \tag{23}$$

$$\begin{aligned} G_{01} = {} & \tfrac{1}{2} n(2VR_{11}/Rr - VR_1/Rr^2 + V_1 R_1/Rr - 2R_{01}/R - 2VR_1\beta_1/Rr) \\ & + [\tfrac{1}{2} n(n-1)/R^2](VR_1^2/r - 2R_0 R_1 - e^{2\beta}) \end{aligned} \tag{24}$$

$$\begin{aligned} G_{22} = {} & (n-1)\, e^{-2\beta}[-VRR_{11}/r + 2RR_{01} + (n-2) R_0 R_1 + VRR_1/r^2 \\ & - RR_1 V_1/r + (1-n/2) VR_1^2/r - (1-n/2)\, e^{2\beta}] \\ & + e^{-2\beta}(2\beta_{01} R^2 - VR^2\beta_{11}/r - R^2\beta_1 V_1/r + VR^2\beta_1/r^2) \\ & + e^{-2\beta} R^2(V_1/r^2 - V/r^3 - V_{11}/2r) \end{aligned} \tag{25}$$

$$\begin{aligned} G_{00} = {} & nR_{00}/R + (nR_0/R)(V_1/2r - V/2r^2 + V\beta_1/r - 2\beta_0) \\ & + nR_1\beta_0 V/rR - nV^2\beta_1 R_1/Rr^2 - nR_1 V_0/2rR - 2nVR_{01}/rR \\ & + nVV_1 R_1/2Rr^2 - nV^2 R_1/2Rr^3 + nV^2 R_{11}/r^2 R \\ & + [n(n-1) V/2rR^2](VR_1^2/r - 2R_0 R_1 - e^{2\beta}) \end{aligned} \tag{26}$$

3. MASS FUNCTION

From the expressions derived in the last section we get

$$R^i_{jij} = 1 + e^{-2\beta}(2R_0 R_1 - VR_1^2/r) \tag{27}$$

Examination of this expression reveals that if $R = r$ and $\beta = 0$, our line element reduces to

$$ds^2 = (1 - R^i_{jij})\, du^2 + 2\, du\, dr - r^2\, dX_n^2 \tag{28}$$

In a recent communication (Chatterjee *et al.*, 1990*b*), we obtained an exterior solution for a charged radiating sphere in $(n+2)$ dimensions for the line element

$$ds^2 = \frac{V}{r}\, du^2 + 2\, du\, dr - r^2\, dX_n^2 \tag{29}$$

The energy-momentum tensor appropriate to this physical situation is given by

$$T_{ab} = \rho U_a U_b + E_{ab} \tag{30}$$

where ρ is the density of radiation and E_{ab} is the electromagnetic energy-momentum tensor. Since the lines of flow are null geodesics and the outflow of radiation is radial,

$$U_a U^a = 0; \qquad U^3 = U^4 = \cdots = U^{n+2}_{n+2} = 0 \tag{31}$$

The Einstein–Maxwell equations via the use of the energy-momentum tensor given above yield the following form of the line element:

$$\begin{aligned} ds^2 = {} & [1 - 2m(u)/(n-1)r^{n-1} + q^2/n(n-1)r^{2n-2}]\, du^2 \\ & + 2\, du\, dr - r^2\, dX_n^2 \end{aligned} \tag{32}$$

When the electromagnetic field is switched off, the solution reduces to a form given recently by Iyer and Vishveshwara (1989) and further when $n=2$ we get the well known Vaidya (1951) metric in the standard four dimensions. So our solution may be termed the electromagnetic generalization of the Vaidya solution in higher dimensions.

Following Israel (1958) and Lichnerowicz (1955), it is obvious that the metric tensor must be continuous across the boundary of a spherical distribution of matter, such that comparing (28) and (32), we get, in the absence of an electromagnetic field,

$$(R^i_{ji1})_b = \frac{2M(u)}{(n-1)r_b^{n-1}} \tag{33}$$

where the evaluation is made at the boundary. We can, at this stage, follow the arguments of McVittie in defining in an ad hoc manner a mass function as

$$\begin{aligned} m(u, r) &= \tfrac{1}{2}(n-1)R^{n-1}[1 + 2R_0 R_1\, e^{-2\beta} - (V/r)R_1^2\, e^{-2\beta}] \\ &= \tfrac{1}{2}(n-1)R^{n-1} R^i_{jij} \end{aligned} \tag{34}$$

Because of the invariance of R^j_{jij} and R, m is an invariant also. The function m may be tentatively interpreted as the total amount of mass-energy entrapped between the center of distribution and the n-space of symmetry. Thus the expression (34) is the higher-dimensional generalization of previous formulations by McVittie *et al.* and also by Hernandez and Misner (1966) to the case when the energy-momentum tensor contains, in addition to a perfect fluid, a radiation. This explains why the mass function depends on the time coordinate u as well.

Let us show now that the mass function is an integral of Einstein's equations. To do that we take for simplicity $i=3$ and $j=2$. From the Bianchi identity

$$R^{3}_{232;a}+R^{3}_{22a;3}+R^{3}_{2a3;2}=0 \tag{35}$$

where a is 1 or 0.

With the identity

$$\Gamma^{2}_{2j}R^{j}_{2a2}=\Gamma^{j}_{22}R^{3}_{ja3} \tag{36}$$

equation (35) reduces to

$$m_a=\tfrac{1}{2}(n-1)[(n-1)R^{n-2}R^{3}_{232}R_a+2R^{n-1}\Gamma^{2}_{2j}R^{j}_{2a2}] \tag{37}$$

From the expression of the Christoffel symbols of our line element and from (18)–(22) we get from (37)

$$m_1=\frac{(n-1)R^n}{n}(G^0_1R_0-G^0_0R_1) \tag{38}$$

$$m_0=\frac{(n-1)R^n}{n}(G^1_0R_1-G^1_1R_0) \tag{39}$$

Using the field equation

$$G_{ij}=-T_{ij} \tag{40}$$

we obtain

$$m_1=\frac{(n-1)R^n}{n}(T^0_0R_1-T^0_1R_0) \tag{41}$$

$$m_0=\frac{(n-1)R^n}{n}(T^1_1R_0-T^1_0R_1) \tag{42}$$

4. APPLICATIONS

Case A. As an application of the expression of the mass function, let us assume that the higher-dimensional spherical ball contains an imperfect fluid with a heat flow in the radial direction but no viscous terms such that the energy-momentum tensor can be written as

$$T^i_j = (\rho + r) U^i U_j - p g^i_j + q^i U_j + q_j U^i \tag{43}$$

where $U^i = dx^i/ds$ are the $(n+2)$-dimensional velocities of the fluid such that for radial flow

$$U^2 = U^3 = U^4 = \cdots = U^n = 0; \qquad U^i U_i = 1 \quad \text{and} \quad q^i U_i = 0; \qquad q^i q_i \geq 0$$

Two viewpoints exist about the nature of the energy-momentum tensor in higher dimensions. We hold the view that the pressure is isotropic in all the dimensions, including the extra ones (Shen and Tan, 1989), in contrast to the view that assumes that the pressure either vanishes in the extra dimensions or has a value which is different from that in the usual 4D space-time.

As discussed earlier (Santos, 1985), the exterior metric that matches with this type of distribution is not the Schwarzschild metric, but the radiating Vaidya form. It is well known that during the gravitational collapse of a star the temperature shoots up to such high values as to cause an intense production of neutrinos. In a pioneering work Misner (1965) studied a nonadiabatic sphere of fluid subject to gravitation which cools via an outward radiation of neutrinos in the radial direction. The neutrinos are neither scattered nor absorbed by the sphere and are represented by a null fluid. The physical picture was later improved upon by Griffiths (1973) and Trim and Wainwright (1971) invoking the concept of a Dirac field.

Here we are not concerned with the physical situation. We use the expression of the mass function to enquire about a possible relationship between the pressure and the heat flow vector existing at the boundary of the spherical distribution. If the equation of the boundary is

$$f(r, u) = 0 \tag{44}$$

then

$$(df/ds)_b = f_1 U^1 + f_0 U^0 = 0 \tag{45}$$

and

$$(dm/du)_b = (-m_1 f_0/f_1 + m_0)_b \tag{46}$$

where $m_0 = (\delta m/\delta u)_r$ and $m_1 = (\delta m/\delta r)_u$.

From (43) it follows that

$$T^0_0 = (\rho + p) U^0 U_0 - p + q^0 U_0 + q_0 U^0 \tag{47}$$

$$T^1_1 = (\rho + p) U^1 U_1 - p + q^1 U_1 + q_1 U^1 \tag{48}$$

$$T_2^2 = T_3^3 = \cdots = T_n^n = -p \tag{49}$$

$$T_0^1 = (\rho + p)U^1 U_0 + q^1 U_0 + q_0 U^1 \tag{50}$$

$$T_1^0 = (\rho + p)U^0 U_1 + q^0 U_1 + q_1 U^0 \tag{51}$$

With the help of equations (41), (42), and (46) we get

$$\begin{aligned}\left(\frac{dm}{du}\right)_b &= \frac{(n-1)R_b^{n-1}}{n(f_1)_b}(T_1^0 R_0 f_0 - T_0^0 R_1 f_0 + T_1^1 R_0 f_1 - T_0^1 R_1 f_1)\\ &= A[\rho(U_1 R_0 - U_0 R_1)(U^0 f_0 + U^1 f_1)\\ &\quad + p(U^1 R_1 + U^0 R_0)(U_1 f_0 - U_0 f_1)\\ &\quad + (U_1 R_0 - U_0 R_1)(q^0 f_0 + q^1 f_1)]\end{aligned} \tag{52}$$

where A is a constant involving the terms outside the bracket.

Invoking the Lichnerowicz continuity conditions, we find that since the mass function is an invariant and depends only on first derivatives, expression (52) should be continuous across the boundary of the distribution.

As mentioned earlier, the exterior energy-momentum tensor corresponding to a radiating sphere is given by

$$T_{ab} = \rho U_a U_b \tag{53}$$

where ρ is the density of radiation. Expression (46) reduces via equation (53) to

$$\left(\frac{dm}{du}\right)_b = \rho U_1 R_0 (U^0 f_0 + U^1 f_1) - \rho U_0 R_1 (U^0 f_0 + U^1 f_1) \tag{54}$$

which, as a result of equation (46), vanishes.

Thus equation (52) yields

$$[p(U^1 R_1 + U^0 R_0)(U_1 f_0 - U_0 f_1) + (U_1 R_0 - U_0 R_1)(q^0 f_0 + q^1 f_1)]_b = 0 \tag{55}$$

For the adiabatic case we thus get the well-known result that $p_b = 0$. In fact equation (56) reestablishes an earlier result of deOliveira *et al.* (1986) that the isotropic pressure on the surface of discontinuity of a spherically symmetric collapsing gaseous distribution with radial heat flow cannot be zero. The pressure will vanish at the boundary only if the fluid is not dissipating, that is, only if there is adiabatic collapse.

Case B. Two viewpoints exist on the significance of extra dimensions for physical processes. Some (Scherk and Schwarz, 1975) regard it as a convenient way of building a complicated Lagrangian in four dimensions proceeding from the simple Lagrangian of gravitation theory in spaces with

$D>4$. The physical meaning of "extra" dimensions remains hidden in such an approach. According to another viewpoint, one postulates the reality of "extra" dimensions which form a compact manifold with the dimension $l_{KK} \sim 10^{-33}$ cm and are unobservable with available experimental facilities. Holding the second viewpoint that extra dimensions are not just a mathematical trick, in this section we utilize the mass function formalism to discuss the case of a spherically symmetric distribution containing both radiation and an electromagnetic field. The exact solution of the metric coefficients given by the present authors is discussed in Section 3. Identifying $R=r$ and comparing (32) and (34), we see that the mass function is given by ($R_0=0$ and $\beta=0$ for the exterior)

$$M(r, u)=m(u)-\frac{q^2}{2nr^{n-1}} \tag{56}$$

when $r=\infty$, $M(\infty, u)=m(u)$, which means that $m(u)$ is the total amount of mass-energy between the center of the distribution and infinity for constant u. This expression is very similar in form to that obtained by Cahill and McVittie with the essential difference that since we here consider a radiating case, the mass terms depend on time coordinate u also. Second, here also the mass function decreases with the decrease of r, but the rate of decrease is much more prominent in the higher-dimensional formalism than in the usual 4D case. Further, a situation may arise when M becomes negative for a very small value of r given by

$$r^{n-1}<\frac{q^2}{2nm(u)} \tag{57}$$

such that a small particle very near a point charge will be repelled. Here also the critical distance at which such a situation would occur depends on the number of dimensions. Equation (56) is interesting in the sense that it is amenable to, in principle at least, observational consequences to check any supposed departure from the usual 4D spacetime. However, it is too premature to come to any definite conclusion in this regard.

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